MTE-01

ASSIGNMENT BOOKLET
Bachelor's Degree Programme
(BS.c./B.A/B.Com)

CALCULUS

Valid from 1st January to 31st December 2014

- **It is compulsory to submit the Assignment before filling in the Term-End Examination Form.**
- **It is mandatory to register for a course before appearing in the Term-End Examination of the course. Otherwise, your result will not be declared.**

For B.Sc. Students Only

- **You can take electives (56 or 64 credits) from a minimum of TWO and a maximum of FOUR science disciplines, viz. Physics, Chemistry, Life Sciences and Mathematics.**
- **You can opt for elective courses worth a MINIMUM OF 8 CREDITS and a MAXIMUM OF 48 CREDITS from any of these four disciplines.**
- **At least 25% of the total credits that you register for in the elective courses from Life Sciences, Chemistry and Physics disciplines must be from the laboratory courses. For example, if you opt for a total of 24 credits of electives in these 3 disciplines, then at least 6 credits out of those 24 credits should be from lab courses.**

School of Sciences
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Maidan Garhi, New Delhi-110068

(2014)

Dear Student,

We hope you are familiar with the system of evaluation to be followed for the Bachelor's Degree Programme. At this stage you may probably like to re-read the section on assignments in the Programme Guide for Elective Courses that we sent you after your enrolment. A weightage of 30 per cent, as you are aware, has been earmarked for continuous evaluation which would consist of **one tutor-marked assignment** for this course.

Instructions for Formatting Your Assignments

Before attempting the assignment please, read the following instructions carefully.

1) On top of the first page of your TMA answer sheet, please write the details exactly in the following format:

ENROLMENT NO: ……

NAME: ……

ADDRESS: ……

……

……

COURSE CODE: …….

COURSE TITLE: …….

ASSIGNMENT NO.: ……..…

STUDY CENTRE: ……..….. **DATE:** …….……...

PLEASE FOLLOW THE ABOVE FORMAT STRICTLY TO FACILITATE EVALUATION AND TO AVOID DELAY.

2) Use only foolscap size writing paper (but not of very thin variety) for writing your answers.

3) Leave 4 cm margin on the left, top and bottom of your answer sheet.

4) Your answers should be precise.

5) While solving problems, clearly indicate the question number along with the part being solved. Be precise. Recheck your work before submitting it.

Answer sheets received after the due date shall not be accepted.

We strongly feel that you should retain a copy of your assignment response to avoid any unforeseen situation and append, if possible, a photocopy of this booklet with your response.

We wish you good luck.

ASSIGNMENT

(To be done **after** studying the course material.)

Course Code : MTE-01
Assignment Code : MTE-01/TMA/2014
Total Marks : 100

Q. 1 Which of the following statements are true? Give reasons for you answer. (This means that if you think a statement is false, give a short proof or an example that shows it is false. If it is true, give a short proof for saying so. For instance, to show that {1, Padma, blue} is a set' is true, you need to say that this is true because it is well defined collection of 3 objectives.)

(i) The domain of the function f given by

$$f(x) = \sqrt{\frac{x^2+2}{x^2-1}} \text{ is } R \setminus \{1\}.$$

(ii) Any curve with portions in the first and second quadrants is symmetric about y-axis.

(iii) $\lim_{x \to 1} \left| \lim_{x \to -1} \frac{2}{x-1} \right|$ does not exist.

(iv) An ever function cannot be monotonic.

(v) The curve $(y^2 - 4) = \frac{1}{(x^2-1)}$ has no asymptote parallel to axes.

(vi) A function f, defined by f(x) = x sin x + cos x, is decreasing in $[o, \pi/2.]$

(vii) If $\int_{-1}^{1} f(x)dx = o,$ then f must be an even function.

(viii) $\frac{d}{dx}\left[\int_{\sqrt{x}}^{x} \sin t^2 dt\right] = \cos x - \cos x^2.$

(ix) $f : \mathbf{R} \to \mathbf{R}$ given by $f(x) = 2|x| - 1$ is differentiable on **R.**

(x) The slope of the tangent of the curve $y = \frac{x}{x^2+2}$ at the origin is 0. (20)

Q.2 (a) Discuss the continuity of the function f, defined by

$$f(x) = \begin{cases} x^2 - 1 & ; x \le 1 \\ 1 - \frac{1}{x} & ; x \ge 1 \end{cases}$$ (3)

(b) Evaluate

(i) $\lim_{x \to 7} \frac{\sqrt{7x} - 7}{\sqrt{(3x-8)} - \sqrt{13}}$

(ii) $\lim_{x \to a} \left(\frac{1}{a-x} - \frac{a}{a^2 - x^2} \right)$ (4)

(c) Find the angle between the x-axis and the tangent to the hyperbola xy = 9 at (3, 3). (3)

Q. 3 (a) Show that f, given by $f(x)=\begin{cases} x\sin\frac{1}{x}, & x\neq 0 \\ 0, & x=0 \end{cases}$ is continuous at each point of **R**. (4)

(b) Check whether the function f: $\mathbf{R}\to\mathbf{R}$ given by $f(x)=|x|-[x]$, is one one onto. Give reasons for your answers. (3)

(c) Find $\lim_{x\to\infty}\frac{5}{x}$. Also, show this graphically. (3)

Q. 4 (a) If $f(x)=\left(\frac{1}{x}\right)^x$, show that $f''(1)=0$. (3)

(b) Find the n^{th} derivative of $\frac{x}{x^2+a^2}$. (3)

(c) If $\cos\left(\frac{y}{b}\right)=\log\left(\frac{x}{n}\right)^n$, then show that $x^2y_{n+2}+(2n+1)xy_{n+1}+2n^2y_n=0$ (4)

Q. 5 (a) Trace the following curves. Give all the properties that are necessary for doing so.

(i) $y^2=(x-1)(x+1)(x-3)$

(ii) $y^2(x-a)=x^2(x+a);a>0.$ (10)

Q. 6 Evaluate the following integrals.

(i) $\int\frac{dx}{1+\cos x}$

(ii) $\int\frac{dx}{(3+4x^2)\sqrt{4-3x^2}}$

(iii) $\int\frac{dx}{\sqrt{4+8x-5x^2}}$

(iv) $\int\frac{dx}{\sqrt{(x-a)(b-x)}},b>a$

(v) $\int_0^{\pi}\frac{x\,dx}{a^2\cos^2x+b^2\sin^2x}.$ (10)

Q.7 (a) If $y_n=\int\frac{\sin nx}{\sin x}dx,$ then show that

$$y=\frac{2\sin(n-1)x}{n-1}+y_{n-2,}$$

Hence, evaluate

$\int_0^{\pi/2} \frac{\sin 7x}{\sin x} dx.$ (5)

(b) show that $x - \frac{x^2}{2} \ln(1+x) < x - \frac{x^2}{2(1+x)} \forall\, x > 0.$ (5)

Q. 8 (a) Find the length of the curve y = In sec x between the points x = 0 and x = π/3. (5)

(b) The lemniscates $r^2 = a^2 \cos 2\theta,$ revolves about a tangent at the pole. Show that the volume generated is $\frac{\pi^2 a^3}{4}$. (5)

Q. 9 (a) Estimate the arc length of $\mathbf{r}(t) = (\cos^2 t, \sin 3t), t \in [0, \pi]$ using trapezoidal rule with n = 6 (3)

(b) Find the area of the region enclosed by the curves $x^2 = y$ and $y = \frac{1}{2}(x^4 + x).$ (3)

(c) Find the maximum and minimum values of the function $f(x) = \sin x + \cos 2x \text{ in } [0, 2\pi].$ (4)