Syllabus for Entrance Test for PHDMT

Section A (50 Marks)

Algebra

Prerequisites and Preliminaries: Logic, Sets and Classes, Functions, Relations and Partitions, Products, The Integers, The Axiom of Choice, Order and Zorn's Lemma.
Groups: Semigroups, Monoids and Groups, Homomorphisms and Subgroups**,** Cyclic Groups, Cosets and Counting, Normality, Quotient Groups, and Homomorphisms, Symmetric, Alternating, and Dihedral Groups, Direct Products and Direct Sums, Free Groups, Free Products, Generators & Relations.
The Structure of Groups: Free Abelian Groups, Finitely Generated Abelian Groups, The Krull-Schmidt Theorem, The Action of a Group on a Set, The Sylow Theorems, Classification of Finite Groups, Nilpotent and Solvable Groups, Normal and Subnormal Series.
Rings: Rings and Homomorphisms, Ideals, Factorization in Commutative Rings, Rings of Quotients and Localization, Rings of Polynomials and Formal Power Series, Factorization in Polynomial Rings.
Fields and Galois Theory: Field Extensions, The Fundamental Theorem, Splitting Fields, Algebraic Closure and Normality, Finite Fields.
Linear Algebra: Vector Space and Linear Transformations, Matrices and Maps, Rank and Equivalence, Determinants, The Characteristic Polynomial, Eigenvectors and Eigenvalues.

Real Analysis

Sequences and series of functions, pointwise and uniform convergence, Cauchy criterion for uniform convergence, Weierstrass M-test, Abel's and Dirichlet's tests for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann-Stieltjes integration, uniform convergence and differentiation, Weierstrass approximation theorem, Power series, uniqueness theorem for power series, Abel's and Tauber's theorems.
Functions of several variables, linear transformations, Derivatives in an open subset of R_n, Chain rule, Partial derivatives, interchange of the order of differentiation, Derivatives of higher orders, Taylor's theorem, Inverse function theorem, Implicit function theorem, Jacobians, extremum problems with constraints, Lagrange's multiplier method, Differentiation of integrals, Partitions of unity, Differential forms, Stoke's theorem.
Lebesgue outer measure. Measurable sets. Regularity. Measurable functions. Borel and Lebesgue measurability. Non-measurable sets.
Integration of Non-negative functions. The General integral. Integration of Series. Reimann and Lebesgue Integrals.
Measures and outer measures, Extension of a measure. Uniqueness of Extension. Completion of a measure. Measure spaces. Integration with respect to a measure. The L_p-spaces. Convex functions, Jensen's inequality. Holder and Minkowski inequalities. Completeness of L_p, Convergence in Measure, Almost uniform convergence.

Topology

Countable and uncountable sets. Infinite sets and the Axiom of Choice. Cardinal numbers and its arithmetic. Schroeder-Bernstein theorem. Cantor's theorem and the continuum hypothesis. Zorn's lemma Well-ordering theorem.
Definition and examples of topological spaces. Closed sets. Closure. Dense subsets.

Neighbourhoods. Interior, exterior and boundary. Accumulation points and derived sets. Bases and sub-bases. Subspaces and relative topology.
Continuous functions and homomorphism, compactness. Continuous functions and compact sets. Basic properties of compactness. Compactness and finite intersection property. Sequentially and countably compact sets. Local compactness and one point compactification. Stone-vech compactification. Compactness in metric spaces. Equivalence of compactness, countable compactness and sequential compactness in metric spaces, Connected spaces (Connectedness only for metric space.)

Functional Analysis

Normed linear spaces. Banach spaces and examples. Quotient space of normed linear spaces and its completeness, equivalent norms. Riesz Lemma, basic properties of finite dimensional normed linear spaces and compactness. Weak convergence and bounded linear transformation, normed linear spaces of bounded linear transformations, dual spaces with examples. Uniform boundedness theorem and some of its consequences. Open mapping and closed graph theorems. Hahn-Banach theorem for real linear spaces, complex linear spaces and normed linear spaces. Reflexive space. Weak Sequential Compactness. Compact Operators. Solvability of linear equations in Banach spaces, the closed Range Theorem.
Inner product spaces. Hilbert spaces. Orthonormal Sets. Bessel’s inequality. Complete orthonormal sets and Parseval’s identity. Structure of Hilbert spaces. Projection theorem. Riesz representation theorem. Adjoint of an operator on a Hilbert space. Reflexivity of Hibert spaces. Self-adjoint operators, Positive, projection, normal and unitary operators. Abstract variational boundary-value problem. The generalized Lax-Milgram theorem.

Differential Equations

Preliminaries-initial value problem and the equivalent integral equation, mth order equation in d-dimensions as a first order system, concepts of local existence, existence in the large and uniqueness of solutions with examples.
Linear Differential Equations-Linear Systems, Variation of constants, reduction to smaller systems. Basic inequalities, constant coefficients. Adjoint systems, Higher order equations.
Dependence on initial conditions and parameters; Preliminaries. Continuity. Differentiability. Higher Order Differentiability.
Linear second order equations-Preliminaries. Basic facts. Theorems of Sturm. Sturm-Liouville Boundary Value Problems. Number of zeros. Nonoscillatory equations and principal solutions. Nonoscillation theorems.
Use of Implicit function and fixed point theorems-Periodic solutions. Linear equations. Nonlinear problems.
Second order Boundary value problems-Linear problems. Nonlinear problems. Aproribounds, Green’s Function.

Partial Differential Equations

Examples of PDE. Classification.
Transport Equation-Initial value Problem. Non-homogeneous Equation.
Laplace’s Equation-Fundamental Solution, Mean Value Formulas, Properties of Harmonic Functions, Green’s Function, Energy Methods.
Heat Equation-Fundamental Solution, Mean Value Formula, Properties of Solutions, Energy methods.
Wave Equation-Solution by spherical Means, Non-homogeneous Equations, Energy Methods.

Nonlinear First Order PDE-Complete Integrals, Envelopes, Characteristics, Hamilton-Jacobi Equations (Calculus of Variations, Hamilton's ODE, Legendre Transform).
Representation of Solutions-Separation of Variables, Similarity Solutions (Plane and Travelling Waves, Solitons, Similarity under Scaling), Fourier and Laplace Transform, Asymptotics (Singular Perturbations, Laplace's Method), Power Series.

Section B (50 Marks)

Research Aptitude

The processes broadly involved in undertaking math research: Ability to generalise and particularise, ability to make 'educated guesses' as conjectures, try to prove/disprove theorems. The objectives are

- To assess the understanding of mathematical research processes.
- To assess the inclination and aptitude for undertaking research in mathematics.